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Photons and Gravitons as Goldstone Bosons, and the Cosmological Constant

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We reexamine a scenario in which photons and gravitons arise as Goldstone bosons associated with the spontaneous breaking of Lorentz invariance. We study the emergence of Lorentz invariant low energy physics in an effective field theory framework, with non-Lorentz invariant effects arising from radiative corrections and higher order interactions. Spontaneous breaking of the Lorentz group also leads to additional exotic but weakly coupled Goldstone bosons, whose dispersion relations we compute. The usual cosmological constant problem is absent in this context: being a Goldstone boson, the graviton can never develop a potential, and the existence of a flat spacetime solution to the field equations is guaranteed.

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1. Introduction

Massless particles can arise by a variety of mechanisms: gauge symmetry, chiral symmetry, supersymmetry, and the spontaneous breaking of global symmetry. The masslessness of the photon and graviton is typically associated with the existence of gauge symmetry; here we explore an alternative option in which it is associated with the spontaneous breaking of Lorentz invariance, with the “gauge” fields arising as Goldstone bosons. This basic idea has a long history, dating back to the 1964 work of Bjorken [1]. For related work see [2,3,4,5].

At the level of low energy effective field theory, the usual argument in favor of exact gauge invariance is based on Lorentz invariance. Some form of gauge invariance is required in order to obtain an interacting, unitary, Lorentz invariant theory of massless particles with spin 1 or 2. In a manifestly Lorentz invariant formulation a violation of gauge invariance will typically imply a noncancellation of timelike and longitudinal modes, yielding a non-unitary S-matrix in the physical sector. In a noncovariant gauge fixed formulation unitarity is manifest, but the action is required to be formally gauge invariant in order to recover Lorentz invariance of the S-matrix.

Especially in the case of gravity, there are various motivations for exploring alternatives. First, it is a basic fact that we inhabit a universe that is not Lorentz invariant at large scales due to cosmological expansion. Second, in the case of gravity gauge invariance is insufficient to guarantee masslessness, since a potential $\Lambda^4\sqrt{-g}$ is allowed. Since a nonzero cosmological constant leads to a non-Lorentz invariant vacuum, the original motivation for gauge invariance is not actually realized.

As an alternative we turn to effective actions that yield massless photons and gravitons as Goldstone bosons of spontaneously broken Lorentz invariance. Of course, the effective theory must be consistent with the observed accuracy of Lorentz invariance at distances small compared to the curvature scale of the universe. This is achieved by effective theories consisting of three sorts of terms: gauge invariant kinetic terms, non-gauge invariant potential terms, and small corrections to these. The simplest example is $\mathcal{L} = (F_{\mu\nu})^2 - V(A^\mu A_\mu)$. As we will discuss, it is not too hard to generate such effective actions from some more conventional underlying dynamics. All terms in the action are taken to be Lorentz invariant; however the potential is assumed to give rise to a constant non-Lorentz invariant vacuum expectation value. The broken Lorentz generators imply the existence of massless Goldstone bosons. In the absence of the potential, the vacuum expectation value would have

no physical effect, being pure gauge; in particular exact Lorentz invariance would be maintained. With the potential included gauge invariance is broken and with it exact Lorentz invariance. However, by definition the Goldstone bosons do not appear in the potential; the expansion of the potential is in terms of massive fields. Since Lorentz invariance is broken only by the potential, the breaking will be suppressed by the inverse mass of the heavy fields. So at low energies we will have an approximately Lorentz invariant theory of Goldstone bosons. The expansion of the potential to quadratic order in fluctuations simply acts a gauge fixing term, so the low energy theory is approximately that of photons or gravitons in a non-covariant gauge.

Actually, there are additional interesting complications in the quantum theory, where loops induce small non-Lorentz invariant kinetic terms. This leads to the appearance of additional non-Lorentz invariant but “weakly coupled” Goldstone bosons whose effects we discuss. Directly related to this is that it is crucial to study whether the form of our effective action is stable under radiative corrections, as these will generically induce all possible operators consistent with the symmetries. We study this question carefully, and find that with reasonable starting assumptions the resulting low energy physics appears approximately Lorentz invariant.

Motivated by the cosmological constant problem, a multitude of authors have experimented with modifying gravity in various ways (see [6] for a review of some attempts). From the point of view of low energy effective field theory the problem is that general covariance allows one – and only one – potential term, $\Lambda^4 \sqrt{-g}$, and so unless this term vanishes there exists no solution to the equations of motion with constant fields. But in a scenario in which the graviton is a Goldstone boson this problem does not arise, since a Goldstone boson can never acquire a potential. So even if some scalar field undergoes a phase transition, contributing a term $V(\phi_0)\sqrt{-g}$ to the effective action, one knows that the vacuum expectation value for massive fields can be shifted such that the Goldstone boson gravitons remain massless. Therefore, there will *always* exist an exact vacuum solution with constant fields, and on which propagate massless gravitons with approximately Lorentz invariant physics. This then guarantees the existence of the sort of solution one wants without fine tuning. However, one should note that there may also exist other solutions with space-time varying fields. A complete solution to the problem should address why a flat (or nearly flat) spacetime solution is preferred; we discuss this in section 3.

The first part of this paper is devoted to the detailed study of the photon as a Gold-

stone boson, but this is essentially a warmup for the more interesting case of gravity, which provides our main motivation. There does not seem to be any obvious advantage in producing the photon as a Goldstone boson, and we have made no attempt at a realistic model by including the other standard model fields. We should also stress that we will work in an effective field theory framework in which the graviton is to be thought of as a composite of more fundamental degrees of freedom. It may be worth mentioning why our scenario is not in conflict with the theorem of Weinberg and Witten [7], which rules out “composite gravitons” in a broad class of models. Specifically, the theorem states that a Lorentz invariant theory with a Lorentz invariant vacuum and a Lorentz covariant energy-momentum tensor cannot have a massless spin two particle in its spectrum. There is no conflict here since our vacuum will not be Lorentz invariant.

This paper is organized as follows. In section 2 we study in detail the example of the photon as a Goldstone boson. The photon example illustrates most of the important general features and is much simpler computationally than the gravity case. We also discuss the relation to previous work on this subject. In section 3 we discuss the graviton and the cosmological constant problem in this context. Section 4 has some final comments, and Appendix A contains technical results for the gravity case.

2. Photon as Goldstone Boson

We begin by writing down a certain effective action for a vector field A_μ coupled to matter. The motivation for this form will be discussed subsequently. The action is to be thought of as an effective action defined at a UV cutoff scale Λ . We thus consider the Lagrangian

$$\mathcal{L} = N \left\{ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - V(A^\mu A_\mu) + \text{higher derivatives} \right\} + \mathcal{L}_{\text{matter}}(\phi, A_\mu) + \mathcal{O}(N^0). \quad (2.1)$$

In the above, N is some large number, and we have written out the leading N terms in the action. The action is Lorentz invariant – all indices being raised and lowered with the Minkowski metric – but not gauge invariant. In particular, the potential $V(A^\mu A_\mu)$ is not gauge invariant; a crucial point is that this is the *only* non-gauge invariant term at leading order in N . In particular, the higher derivatives terms are terms like $(F^{\mu\nu} F_{\mu\nu})^2$, $\partial^\alpha F^{\mu\nu} \partial_\alpha F_{\mu\nu}$, etc. Similarly, the generic matter fields ϕ are gauge invariantly coupled to A_μ in $\mathcal{L}_{\text{matter}}(\phi, A_\mu)$. Apart from these stipulations the action is generic in the sense that

all dimensionless couplings (apart from N) are of order unity; that is, all dimensionful quantities are of the order the cutoff Λ to the appropriate power.

Since in our effective action we have explicitly excluded various terms which are consistent with the symmetries of the theory, two questions immediately come to the fore: how might an action of this form be generated from some underlying dynamics, and is the assumed structure of the action stable under radiative corrections? We now address these two points in turn.

2.1. Effective action from fermions

To show why an action of the form (2.1) is fairly natural, we show how it can be generated by integrating out some large number of fermion species. This will be a generalization of the original mechanism proposed by Bjorken [1], which considered four fermion interactions. So consider N species of Dirac fermions ψ_i . We imagine these fermions being coupled to gauge fields which acquire masses at scale Λ . Integrating out the massive gauge bosons will yield an infinite set of fermion interactions, and we will focus on the following subset:

$$\mathcal{L}_\psi = \bar{\psi}_i(i\cancel{\partial} - m)\psi_i + N \sum_{n=1}^{\infty} \lambda_{2n} \frac{(\bar{\psi}_i \gamma^\mu \psi_i)^{2n}}{N^{2n}}. \quad (2.2)$$

Here, summation over flavor indices i and spacetime indices μ is implied. We wrote the action to have a $U(N)$ flavor symmetry. The couplings λ_{2n} are of order unity times the appropriate power of Λ ,

$$\lambda_{2n} \sim \Lambda^{4-6n}. \quad (2.3)$$

Factors of N have been inserted in order to give a well defined large N limit. In particular, the normalized bilinears $\mathcal{O}^\mu = \frac{1}{N} \bar{\psi}_i \gamma^\mu \psi_i$ then have correlators scaling as N^0 , and the action written in terms of \mathcal{O} has an overall factor of N .

We will employ the standard trick of rendering the action quadratic in fermions by introducing an auxilliary field A^μ . We therefore consider

$$\mathcal{L}_{\psi,A} = \bar{\psi}_i(i\cancel{\partial} - \cancel{A} - m)\psi_i - NV(A^\mu A_\mu). \quad (2.4)$$

The potential V is a power series in $A^\mu A_\mu$ with coefficients chosen such that by solving the algebraic equations of motion for A^μ and substituting back in we recover (2.2). The most familiar case corresponds to a pure four fermion interaction, with only λ_2 nonvanishing, in which case $V(A^\mu A_\mu) = A^\mu A_\mu / 4\lambda_2$. The quantum version of this theory is defined by

a path integral (with a cutoff) over the fields ψ_i and A^μ . The idea is to imagine first doing the path integral over ψ_i to yield an effective action for A^μ . Since ψ_i is minimally coupled to A^μ , provided we choose a gauge invariant cutoff the terms in the effective action generated in this way will be gauge invariant. Furthermore, since there are N species of fermions the effective action will have an overall factor of N . Therefore, the form of the effective action is that of the first set of terms in (2.1).

Now suppose we had included the other fermion terms in (2.2) that would certainly arise upon integrating out massive gauge fields. This will introduce other bilinears in the theory, *e.g.*, $\bar{\psi}_i \gamma^{\mu\nu} \psi_i$, $\bar{\psi}_i \gamma^\mu \partial^\nu \psi_i$, etc. The above procedure should then be generalized by introducing a new auxilliary field for each bilinear. Integrating out the fermions then yields an effective action for a set of interacting auxilliary fields. The analysis rapidly becomes complicated; however, one expects on general grounds that the auxilliary fields will acquire mass terms of order of the cutoff and so can be neglected at lower energies. By contrast, in our scenario certain components of A^μ will remain massless since they will correspond to Goldstone bosons.

To reiterate somewhat, in our approach where we consider A^μ as the only auxilliary field it is consistent to omit terms like $(\bar{\psi}_i \gamma^\mu \psi_i)^2 \bar{\psi}_j \not{\partial} \psi_j$ which might seem to lead to non-gauge invariant terms like $f(A^\mu A_\mu) F^{\alpha\beta} F_{\alpha\beta}$ in the effective action for A^μ . If such fermionic terms are to be included one should introduce a new scalar auxilliary field for the bilinear $\bar{\psi}_i \not{\partial} \psi_j$, and we are not doing this for the reasons stated above.

We have thus demonstrated one possible way of generating an effective action with the structure (2.1), though there are presumably other ways as well. For the most part we consider (2.1) in its own right, without reference to its origin.

2.2. Coupling to matter

By coupling a matter field ϕ to the fermions via conserved currents, $J^\mu(\phi) \bar{\psi}_i \gamma_\mu \psi_i$, we will generate the matter couplings given in (2.1). An easy way to accomplish this is to modify (2.2) by taking the mass for some of the fermions to be much less than the cutoff Λ . In this case we would keep the light fermions in the low energy effective action rather than integrating them out. From (2.4) we see that these fermions will be minimally coupled to A^μ . Whether we use this or some other mechanism to generate the matter couplings, it will be important that the matter action is gauge invariant, at least up to the level of two derivative terms.

2.3. Stability under radiative corrections

Treating (2.1) as an effective field theory at scale Λ , to extract out low energy physics we still need to integrate out the fluctuations of A^μ . Since the potential term violates gauge invariance, once we start computing loop diagrams all possible Lorentz invariant, but not necessarily gauge invariant, terms will generically be generated. Some of these terms would lead, after spontaneous symmetry breaking, to large violations of Lorentz invariance at low energies and so need to be suppressed. The need to suppress such terms is our motivation for introducing the large number N . Given the form (2.1), the loop expansion is an expansion in $1/N$, so the dangerous terms will only arise at order N^0 .

So at order N^0 we need to consider all possible Lorentz invariant terms generated by computing loop diagrams for A^μ . At energies low compared to the cutoff we can restrict attention to terms with at most two derivatives. We further assume symmetry under charge conjugation C , acting as sign reversal on A^μ . This forbids single derivative terms. Terms with no derivatives just give a small correction to the potential in (2.1), which we are taking to be arbitrary, so we need consider only two derivative terms. Up to integration by parts, there are seven independent terms:

$$\begin{aligned}
& 1) \ f_1(A^2) \partial_\mu A_\nu \partial^\mu A^\nu \\
& 2) \ f_2(A^2) \partial_\mu A_\nu \partial^\nu A^\mu \\
& 3) \ f_3(A^2) A^\mu A^\alpha \partial_\mu A_\nu \partial_\alpha A^\nu \\
& 4) \ f_4(A^2) A^\nu A^\alpha \partial_\mu A_\nu \partial_\alpha A^\mu \\
& 5) \ f_5(A^2) A^\nu A^\alpha \partial_\mu A_\nu \partial^\mu A_\alpha \\
& 6) \ f_6(A^2) A^\mu A^\nu A^\alpha \partial_\mu \partial_\nu A_\alpha \\
& 7) \ f_7(A^2) A^\mu A^\nu A^\alpha A^\beta \partial_\mu A_\nu \partial_\alpha A_\beta
\end{aligned} \tag{2.5}$$

Here $A^2 \equiv A^\mu A_\mu$. As always, we assume that all dimensionful couplings in f_i are of order unity times the appropriate power of Λ . As we will see, after spontaneous symmetry breaking some of these terms will lead to low energy violations of Lorentz invariance at order $1/N$.

2.4. Spontaneous symmetry breaking

Our potential will generically have the form

$$V(A^2) = \Lambda^4 \sum_{n=1}^{\infty} V_n \left(\frac{A^2}{\Lambda^2} \right)^n, \tag{2.6}$$

with the coefficients V_n of order unity. V_n can be determined in terms of the λ_{2n} appearing in (2.2). We will assume that the potential leads to spontaneous symmetry breaking,

$$\langle A_\mu \rangle = c\Lambda n_\mu, \quad (2.7)$$

where for definiteness n_μ is a spacelike unit vector and c is of order unity. This expectation value spontaneously breaks Lorentz invariance at the cutoff scale Λ . Nevertheless, we will see that low energy physics is approximately Lorentz invariant.

We now expand the action around the vacuum (2.7) by writing

$$A_\mu = c\Lambda n_\mu + a_\mu. \quad (2.8)$$

To quadratic order in a_μ the potential becomes

$$V = V(-c^2\Lambda^2) + \frac{1}{2\alpha}(n \cdot a)^2 + \dots, \quad (2.9)$$

where

$$\alpha = \frac{1}{4V''(-c^2\Lambda^2)} \sim \frac{1}{\Lambda^2}. \quad (2.10)$$

Shifting the vacuum has a trivial effect on the A^μ kinetic terms since, being gauge invariant, these depend only on derivatives of A^μ . Hence in the kinetic terms we can just replace $A^\mu \rightarrow a^\mu$. We can similarly make this replacement in the matter Lagrangian after performing a compensating gauge rotation of the matter fields:

$$\mathcal{L}_{\text{matter}}(\phi, \langle A_\mu \rangle + a_\mu) = \mathcal{L}_{\text{matter}}(\phi', a_\mu), \quad (2.11)$$

where ϕ' is a gauge transformation of ϕ . We will henceforth drop the prime on ϕ . Therefore, the action takes the form

$$\mathcal{L} = N \left\{ -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\alpha}(n \cdot a)^2 + \text{higher derivatives} + \mathcal{O}(a^3) \right\} + \mathcal{L}_{\text{matter}}(\phi, a_\mu) + \mathcal{O}(N^0). \quad (2.12)$$

The $(n \cdot a)^2$ term plays the role of an axial gauge fixing term. It gives a mass of order Λ to a spacelike component of a_μ . So to the above order our action takes the form of an axial gauge fixed photon coupled gauge invariantly to matter. Neglecting the higher derivative terms, the photon propagator is given by

$$-\frac{i}{Np^2} \left(\eta_{\mu\nu} - \frac{1}{n \cdot p}(n_\mu p_\nu + n_\nu p_\mu) - \frac{p_\mu p_\nu}{(n \cdot p)^2}(\alpha p^2 - n^2) \right). \quad (2.13)$$

As usual, only the first term contributes when the propagator is sandwiched between conserved current, and the result is Lorentz invariant. Corrections to this Lorentz invariant result are suppressed by p/Λ and/or $1/N$, as will be discussed below.

2.5. Goldstone bosons

After spontaneous symmetry breaking we have “massless particles”¹ by virtue of Goldstone’s theorem. In particular, a spacelike vector breaks the Lorentz group according to

$$SO(3, 1) \longrightarrow SO(2, 1). \quad (2.14)$$

There are thus three broken generators corresponding to two rotations and a boost. The corresponding three Goldstone bosons are the three components of a^μ orthogonal to n^μ . We choose a basis of two transverse components and a timelike component:

$$\begin{aligned} \text{transverse : } & \epsilon_\mu^{(1,2)}, \quad k \cdot \epsilon^{(1,2)} = n \cdot \epsilon^{(1,2)} = 0 \\ \text{timelike : } & \epsilon_\mu^{(0)} = k_\mu + (n \cdot k)n_\mu, \quad \text{obeys : } n \cdot \epsilon^{(0)} = 0. \end{aligned} \quad (2.15)$$

At low energies only the Goldstone bosons are relevant. At leading order in N the gauge invariant form of the kinetic terms implies that only the transverse Goldstone bosons propagate, giving us the conventional Lorentz invariant electrodynamics. However, at order N^0 the timelike component will also propagate, and this will lead to interesting effects.

We now concentrate on the low energy physics of the Goldstone bosons coupled to matter. The Goldstone bosons can be thought of as coordinates on the coset $SO(3, 1)/SO(2, 1)$. Since the Goldstone bosons label flat directions of the potential, the cubic and higher order terms from the expansion of the potential all involve the massive component $n \cdot a$, and so are irrelevant at low energies. Now consider the order N^0 terms (2.5). Note that after spontaneous symmetry breaking the terms (4) - (7) expanded to quadratic order in fluctuations will all involve at least one factor of $n \cdot a$. Therefore, only terms (1) - (3) are relevant for low energy physics. Furthermore, one linear combination of (1) and (2) is proportional to $(F_{\mu\nu})^2$ and so just provides a small correction to the order N value of this term. Hence we can omit one linear combination, say (2), and focus only on terms (1) and (3). It is also convenient at this point to rescale $a^\mu \rightarrow a^\mu/\sqrt{N}$ to put the gauge kinetic term in standard form. Therefore to order N^0 , and discarding terms suppressed at low energies by p/Λ , the effective action is

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\alpha}(n \cdot a)^2 + \frac{1}{2}\frac{c_1}{N}\partial^\mu a^\nu \partial_\mu a_\nu + \frac{1}{2}\frac{c_2}{N}n^\alpha n^\beta \partial_\alpha a_\mu \partial_\beta a^\mu + \mathcal{L}_{\text{matter}}(\phi, a_\mu/\sqrt{N}). \quad (2.16)$$

¹ We employ quotation marks since we can no longer use the standard definition of particles as being irreducible representations of the (now spontaneously broken) Poincare group. But it should be clear what we mean when we use the particle terminology.

Here we defined the order unity numerical coefficients $c_{1,2}$ as

$$f_1(\langle A \rangle^2) = \frac{1}{2}c_1, \quad c^2\Lambda^2 f_3(\langle A \rangle^2) = \frac{1}{2}c_2. \quad (2.17)$$

We also kept the term $(n \cdot a)^2$ as a convenient way of implementing the axial gauge condition.

(2.16) clearly shows the need for a $1/N$ suppression of the third and fourth terms in order to have approximate Lorentz invariance at low energies. It is easiest to compute the propagator from the first two terms and to think of the third and fourth terms as interactions. Then it is clear that when we compute interaction between conserved matter currents we will get the standard QED results at leading order, with non-Lorentz invariant corrections occurring at order $1/N$.

2.6. Low energy spectrum

We now determine the dispersion relations for the three Goldstone bosons by solving the linearized equations of motion. The latter are

$$\left(1 + \frac{c_1}{N}\right) \partial^\mu \partial_\mu a^\nu - \partial^\nu \partial_\mu a^\mu + \frac{c_2}{N} n^\alpha n^\beta \partial_\alpha \partial_\beta a^\nu - \frac{1}{\alpha} n^\mu a_\mu n^\nu = 0. \quad (2.18)$$

First consider the transverse modes. Plugging in the ansatz (see (2.15))

$$a_\nu = \epsilon_\nu^{(1,2)} e^{-ik \cdot x} \quad (2.19)$$

we find the dispersion relation

$$\text{transverse : } \left(1 + \frac{c_1}{N}\right) k^2 + \frac{c_2}{N} (n \cdot k)^2 = 0. \quad (2.20)$$

This corresponds to an anisotropic speed of light. The speed of light parallel to n^μ differs from that orthogonal to n^μ by an amount of order $1/\sqrt{N}$.

Now consider the timelike mode. We plug in the ansatz

$$a_\nu = (k_\nu + \gamma n_\nu) e^{-ik \cdot x}. \quad (2.21)$$

We find

$$\gamma = n \cdot k + \mathcal{O}(\alpha k^2), \quad (2.22)$$

and the dispersion relation

$$\text{timelike : } k^2 - \frac{N}{c_1} (n \cdot k)^2 = 0. \quad (2.23)$$

In the dispersion relation we have dropped terms down by $1/N$ or αk^2 . The dispersion relation (2.23) is non-Lorentz invariant at leading order. The timelike modes propagate at the ordinary speed of light in directions orthogonal to n^μ , but at a speed of order \sqrt{N} in the direction parallel to n^μ . We are assuming that $c_1 > 0$.

The physics of the transverse modes is thus standard up to small corrections, while that of the timelike mode is quite exotic. The reason for this is that the timelike mode does not propagate with respect to the leading N gauge invariant kinetic terms of (2.1). It only acquires a kinetic term at order N^0 , and these terms are non-Lorentz invariant after spontaneous symmetry breaking.

While exotic, the timelike mode leads to acceptably small effects for sufficiently large N . Its contribution to the interaction between conserved currents is suppressed by $1/N$, since as we have discussed we can use the standard axial gauge propagator at leading order and regard corrections as coming from interaction vertices with coefficients of order $1/N$.

The timelike mode is also suppressed by phase space considerations. For fixed available energy k^0 the dispersion relation forces

$$|k \cdot n| < \sqrt{\frac{c_1}{N}} k^0. \quad (2.24)$$

Consider putting the system in a box of size L . Then for $N > (k^0 L)^2$ only the zero momentum mode parallel to n^μ survives, yielding a phase space suppression proportional to $1/L$.

While the timelike mode gives small corrections to the interaction between conserved currents it could have more dramatic consequences given its unusual dispersion relation. We should emphasize that the result (2.23) does not necessarily imply faster than light² signal propagation, since (2.23) is only valid for long wavelengths. Also, even if (2.23) could be extrapolated to short wavelengths so that signals could propagate faster than light, there would be no conflict with causality since Lorentz invariance has been spontaneously broken by a preferred frame. It would be interesting to study the physics of the timelike mode in more detail.

2.7. Summary and relation to previous work

Let us summarize what has been accomplished. We have shown that spontaneous breaking of Lorentz symmetry can lead to an approximately Lorentz invariant low energy

² Here we refer to the speed of light as the speed of the transverse modes after spontaneous symmetry breaking.

theory of massless photons coupled to matter. This is possible in the context of a theory in which gauge invariance is violated at leading order only by a potential term. Since the Goldstone boson photons do not appear in the potential, the Lorentz violating condensate leads to only small corrections to the low energy physics of the photons. On the other hand, the existence of three broken Lorentz generators implies the existence of a third Goldstone boson whose physics is not even approximately Lorentz invariant. However, its effects are suppressed by $1/N$ and phase space considerations. Altogether, Lorentz invariance appears as an approximate symmetry of the low energy world.

This is a good place to compare and contrast with previous work on this subject, in particular the original work of Bjorken [1]. The main difference is that we have taken a modern effective field theory point of view, emphasized that the violation of Lorentz invariance is real, and pointed out the existence of an extra Goldstone boson. Earlier work started from a four-fermi interaction, *i.e.* just keeping the term λ_2 in (2.2). The trouble with this is that it is incompatible with spontaneous symmetry breaking, since it corresponds to a potential $V \sim A^\mu A_\mu$ with no higher order terms. A reflection of this is that the condensate was never actually computed in earlier work, but was either *assumed* to arise somehow, or emerged after formal manipulations with divergent integrals. The claim was then made that the physics after spontaneous symmetry breaking was the usual *exactly* Lorentz invariant quantum electrodynamics. This conclusion was again dependent on manipulating divergent quantities. As far as we can tell, the origin of this claim is that if one takes the pure four fermi interaction and assumes (wrongly) that this leads to spontaneous symmetry breaking, then expanding $V \sim A^\mu A_\mu$ around the new vacuum would yield an axial gauge fixing term and nothing more. This of course gives usual QED in axial gauge. But from our point of view it is clear what would actually happen in this theory. The four fermi theory either leads to an instability or to a stable vacuum with a massive vector field A_μ . In neither case does one find QED. On the other hand, the problem disappears once one includes the higher order fermion terms as we have done; this is also the natural starting point from the view of effective field theory.

Since earlier work took the point of view that the spontaneous breaking of Lorentz invariance was somehow fictitious, the existence of the extra Goldstone boson was not noted.

Before turning to gravity we should also note that some of the above criticisms were commented on recently by Bjorken [8]. In particular it was noted that the four fermi

theory by itself is inadequate, and that some real violations of Lorentz invariance should be expected once quantum effects are taken into account. These points were also examined by Banks and Zaks in [9] in the context of non-abelian gauge fields; they also concluded that there is a real violation of Lorentz invariance. We hope to have resolved these issues here.

3. Graviton as Goldstone boson

We now turn to our main interest: producing a graviton as a Goldstone boson. Fortunately, the analysis closely parallels the photon case, and so we can draw on our experience from that example to navigate in the more complicated gravitational setting. The main difference is in the different pattern of spontaneous Lorentz breaking as well as in the connection to the cosmological constant problem. The cosmological constant provides a motivation for modifying the low energy effective theory of general relativity, and indeed we will see that the problem is avoided in the sense that the Goldstone boson graviton remains massless even in the presence of vacuum energy.

To adapt the previous approach to the case of gravity we consider an effective action in direct analogy with (2.1),

$$\mathcal{L} = N \{ \Lambda^2 \sqrt{-g} R(g) - \Lambda^4 V(h) + \text{higher derivatives} \} + \mathcal{L}_{\text{matter}}(\phi, g) + \mathcal{O}(N^0). \quad (3.1)$$

Here h is defined via expansion of the metric around flat spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \quad (3.2)$$

Flat spacetime thus plays a preferred role in this context; indeed we think of the underlying dynamics as that of a nongravitational theory in Minkowski spacetime. Appendix A discusses one possibility for such an underlying theory. The Einstein-Hilbert term in (3.1) is standard while the potential is some generic Lorentz invariant function of $h_{\mu\nu}$ with indices contracted with $\eta_{\mu\nu}$. Odd powers of h are allowed, for instance the term h^μ_μ can appear in the expansion of $V(h)$. As in the photon case the higher derivative terms are generally covariant, as is the matter action. General covariance is violated only by the potential.

Note that the observed Newton's constant will be

$$G_N \sim \frac{1}{N\Lambda^2}. \quad (3.3)$$

Therefore, the cutoff Λ is smaller by a factor of $1/\sqrt{N}$ compared to the usual Planck scale. Indeed, this lowered value of the cutoff is responsible for suppressing loop corrections. N is a large number but presumably need not be more than 10^4 or so. Thus the cutoff can still be well above observable energy scales.

We consider potentials leading to a vacuum expectation value for $h_{\mu\nu}$. By performing a Lorentz transformation we can bring the expectation value to the form

$$\langle h_{\mu\nu} \rangle = \begin{pmatrix} \bar{h}_{00} & & & \\ & \bar{h}_{11} & & \\ & & \bar{h}_{22} & \\ & & & \bar{h}_{33} \end{pmatrix}. \quad (3.4)$$

For $\bar{h}_{\mu\mu}$ all nonvanishing and distinct, the Lorentz group will be completely broken:

$$SO(3,1) \longrightarrow \text{nothing}. \quad (3.5)$$

Being dimensionless, we expect $\bar{h}_{\mu\mu}$ of order unity. Therefore there will be 6 Goldstone bosons corresponding to the six broken Lorentz generators. The Lorentz generator $J^{\mu\nu}$ acts on $\langle h_{\mu\nu} \rangle$ by exciting the $\mu \neq \nu$ components. So the Goldstone bosons are the six off-diagonal components of the symmetric matrix $h_{\mu\nu}$. Fluctuations of the diagonal components will generically correspond to massive fields. We will ultimately associate two of the Goldstone bosons with the two physical polarizations of the graviton, while the remaining four will appear in analogy with the timelike Goldstone boson in the photon case.

We now consider fluctuations around the vacuum by writing

$$h_{\mu\nu} = \langle h_{\mu\nu} \rangle + \tilde{h}_{\mu\nu}. \quad (3.6)$$

The expansion of the potential will correspond to mass terms for the diagonal components of $\tilde{h}_{\mu\nu}$. The precise form of this mass matrix is not important, so we will write

$$V(h) = \text{constant} + \Lambda^4 \sum_{\alpha=0}^3 (f_\alpha n_{(\alpha)}^\mu n_{(\alpha)}^\nu \tilde{h}_{\mu\nu})^2 + \mathcal{O}(\tilde{h}^3). \quad (3.7)$$

Here

$$n_{(\alpha)}^\mu = \delta_\alpha^\mu, \quad (3.8)$$

and f_α are numbers of order unity.

We can simplify the action by performing a general coordinate transformation to put the background metric back in standard form. In particular, introduce new coordinates x'^μ such that

$$\frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial x'^\nu}{\partial x^\beta} \eta_{\mu\nu} = \eta_{\alpha\beta} + \langle h_{\alpha\beta} \rangle. \quad (3.9)$$

The metric appearing in the action will then be $\eta_{\mu\nu} + \tilde{h}'_{\mu\nu}$ where

$$\frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial x'^\nu}{\partial x^\beta} \tilde{h}'_{\mu\nu}(x') = \tilde{h}_{\alpha\beta}(x). \quad (3.10)$$

We similarly act with a coordinate transformation on the matter fields; *e.g.* for a scalar

$$\phi'(x') = \phi(x). \quad (3.11)$$

After changing the integration variable to x' the potential term is modified while the general covariant terms are of course invariant. Given the form (3.10), the modification of the potential can be absorbed in a redefinition of the constants f_α . So after the coordinate transformation our action takes the form

$$\mathcal{L} = N \left\{ \Lambda^2 \sqrt{-g} R(g) - \Lambda^4 V(\tilde{h}') + \text{higher derivatives} \right\} + \mathcal{L}_{\text{matter}}(\phi', g) + \mathcal{O}(N^0), \quad (3.12)$$

with

$$g_{\mu\nu} = \eta_{\mu\nu} + \tilde{h}'_{\mu\nu} \quad (3.13)$$

and

$$V(\tilde{h}') = \text{constant} + \Lambda^4 \sum_{\alpha=0}^3 (f_\alpha n_{(\alpha)}^\mu n_{(\alpha)}^\nu \tilde{h}'_{\mu\nu})^2 + \mathcal{O}(\tilde{h}'^3). \quad (3.14)$$

We henceforth relabel fields: $\tilde{h}' \rightarrow h$, $\phi' \rightarrow \phi$.

We think of the quadratic terms in the potential as gauge fixing terms, corresponding to the gauge

$$g_{\mu\mu} = \eta_{\mu\mu}, \quad \text{no sum.} \quad (3.15)$$

This defines an acceptable noncovariant gauge. The graviton propagator in this gauge is extremely complicated and unwieldy, and so we will not display it here. Fortunately, for leading order calculations we only need to know that it has the structure of the standard covariant propagator

$$\frac{-i}{p^2} (\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha} - \eta_{\mu\nu} \eta_{\alpha\beta}) \quad (3.16)$$

plus terms with at least one factor of p (vector) in the numerator. Sandwiched between conserved energy momentum tensors the latter p terms vanish, and so we recover the standard Lorentz invariant result. Of course this is no surprise, since (3.15) represents a valid gauge choice.

The low energy physics is therefore quite similar to what we found in the photon example. We will have two graviton states which propagate at the speed of light, up to a small anisotropic correction. Further there are four additional Goldstone bosons that acquire kinetic terms at order $1/N$. These will have highly non-Lorentz invariant dispersion relations, but their couplings to conserved currents are suppressed by $1/N$. Working out these dispersion relations explicitly would be quite involved given the large number of terms in the action at order $1/N$ and the proliferating indices. However from our discussion of the photon example it should be clear that the essential physics is independent of these details.

3.1. The cosmological constant

Notice that we have obtained an approximately Lorentz invariant theory of gravity without making any specific assumptions about the form of the potential $V(h)$. Therefore, we see that if the potential is suddenly modified, say by a matter phase transition, then the vacuum expectation value of h can simply shift to the new minimum, leaving us again with an approximately Lorentz invariant theory. In particular, the term $\sqrt{-g}V_{\text{matter}}(\phi_0)$ can be added to our previous potential and the analysis proceeds as before. We have therefore evaded the usual cosmological constant problem. The usual problem arises because of general covariance: only a single potential term is allowed, $\Lambda^4\sqrt{-g}$, and a nonzero value of this term is incompatible with a Lorentz invariant solution. If one is willing to violate general covariance by writing a more general potential then this conclusion need not follow. Indeed, if the graviton is a Goldstone boson one is guaranteed to find a solution with constant fields and a massless graviton. What is perhaps surprising is that the physics around such a non-Lorentz invariant solution is approximately Lorentz invariant, as we have seen.

On the other hand, the above discussion does not immediately imply that the approximately Lorentz invariant solutions are the *only* solutions. Indeed, at least in the weak field approximation we will find additional approximately de Sitter and anti-de Sitter solutions. This follows from the fact that at low energies and for weak fields our theory is that of standard gravity in a noncovariant gauge, plus additional weakly coupled Goldstone

bosons. If one now takes a standard solution with a given energy momentum tensor and expresses it in our gauge, this must continue to be an approximate solution of our theory. The existence of multiple solutions is hardly surprising given that we are solving (at leading order) second order differential equations³. Note that the approximately Lorentz invariant solution with constant fields is guaranteed to be an exact solution as it just corresponds to extremizing the potential, while the other solutions with spacetime varying fields need not be exact. In a more conventional field theory context one would expect a solution with time dependent fields to eventually settle down to a static solution by radiating energy. One might expect the same here, with the time dependent fields of the de Sitter type solutions radiating away leading to the solution with constant fields. A realistic proposal in this framework must involve showing how to make the transition from an expanding radiation or matter dominated universe to the approximately Lorentz invariant solution discussed above. We hope to return to this in future work.

4. Concluding Remarks

Building on the work of Bjorken, we have shown that massless photons and gravitons can be produced as Goldstone bosons associated with the spontaneous breaking of Lorentz invariance, and with low energy physics appearing Lorentz invariant to high accuracy. The most dramatic effect of the Lorentz breaking is the existence of additional weakly coupled Goldstone bosons obeying highly non-Lorentz invariant dispersion relations. These fields would be difficult to detect as they couple weakly to conserved currents. A rather general framework for studying Lorentz violating extensions of the Standard Model has been developed (see, *e.g.*, [12]), and it might be useful to study some of our results in that language.

We find it interesting that the observed low energy physics of gravity can be produced in the context of an effective field theory that differs markedly from general relativity, and which does not suffer from the usual cosmological constant problem. While it remains to be seen whether a theory of this type could be incorporated into a truly fundamental framework or be developed into a realistic cosmology, it seems to be an idea worth pursuing.

³ It is also reminiscent of brane world scenarios for addressing the cosmological constant problem [10,11]. But there the Lorentz invariant solution on the brane is tied up with a naked singularity away from the brane, and so need not exist.

Appendix A.

In this appendix we indicate one way the gravitational effective action (3.1) may emerge from some underlying conventional dynamics. There may be others. Here we employ the same mechanism as in the photon case, (2.1). Thus we consider N fermions coupled to gauge fields that acquire masses. We then imagine integrating out the fields from some initial Λ_0 down to a scale Λ obtaining the effective action

$$\bar{\psi}_i (i \not{\partial} - M) \psi_i + 4\pi^2 N \sum_k C_k(\Lambda_0, \Lambda) \mathcal{O}_k(\Lambda) \quad (\text{A.1})$$

involving an infinite set of fermion interactions. Restricting first to the subset consisting only of powers of

$$\mathcal{O}_{\mu\nu} = \frac{1}{N} \bar{\psi}_i \frac{i}{2} \left(\gamma_\mu \vec{\partial}_\nu - \gamma_\mu \overleftarrow{\partial}_\nu \right) \psi_i, \quad (\text{A.2})$$

we introduce the symmetric auxiliary field $h^{\mu\nu}$ to render the effective action quadratic in the fermions, so that we may write it in the form (cp. (2.4)):

$$\mathcal{L}_{\psi,h} = (\eta^{\mu\nu} + h^{\mu\nu}) \bar{\psi}_i \frac{i}{2} \left(\gamma_\mu \vec{\partial}_\nu - \gamma_\mu \overleftarrow{\partial}_\nu \right) \psi_i - M \bar{\psi}_i \psi_i - N \frac{\Lambda^4}{4\pi^2} V(h). \quad (\text{A.3})$$

All indices are raised and lowered by the flat metric $\eta_{\mu\nu}$.

Integrating out the fermions, the effective action from the resulting determinant can be expressed, as usual, as the sum over all fermion 1-loop diagrams with external h legs. Note, in particular, that the diagram with one external h -leg is in general nonvanishing. This reflects the fact that (A.2) has a nonvanishing expectation (proportional to $\eta_{\mu\nu}$) even in ordinary perturbation theory on a Lorentz invariant vacuum, i.e. interactions built from (A.2) shift the classical background. Correspondingly, all terms, including a linear term, are included in the general potential $V(h)$ in (3.1) (cp. discussion in the text).

Explicit evaluation of the fermion-loop graphs with one and two external h -legs gives, after a lengthy computation, the contribution to the effective action to $\mathcal{O}(h^2)$:

$$\begin{aligned} \mathcal{L}^{(2)} = & N I_4 \left[-h_\mu^\mu + \frac{1}{2} h^{\mu\nu} h_{\mu\nu} + \frac{1}{2} (h_\mu^\mu)^2 \right] \\ & + \frac{N}{6} I_2 \left[(\partial_\lambda h^{\mu\nu})^2 - (\partial_\nu h_\mu^\mu)^2 + 2 \partial_\mu h^{\mu\nu} \partial_\nu h_\lambda^\lambda - 2 (\partial_\mu h^{\mu\nu})^2 \right] \\ & - \frac{N}{20} I_0 \left[(\Box h^{\mu\nu})^2 - \frac{1}{3} (\Box h_\mu^\mu)^2 - 2 (\partial_\mu \partial_\lambda h^{\lambda\nu})^2 + \frac{2}{3} \partial_\mu \partial_\nu h^{\mu\nu} \Box h_\lambda^\lambda \right. \\ & \quad \left. + \frac{2}{3} (\partial_\mu \partial_\nu h^{\mu\nu})^2 \right] + \dots \end{aligned} \quad (\text{A.4})$$

(A.4) displays explicitly the leading terms, i.e. the local, cut-off dependent (‘divergent’) part of the result of the loop integrations. The ellipses denote the subleading pieces from the finite, non-local parts, which can be expanded in powers of \Box/M^2 , and contribute to higher derivative interactions relevant only for short distance behavior near the cutoff. Dimensional regularization, under the usual correspondence $\ln \Lambda \leftrightarrow (1/\epsilon + \text{const.})$, gives

$$I_n = \frac{1}{(4\pi)^2} M^n \ln\left(\frac{\Lambda^2}{M^2}\right), \quad n = 0, 2, 4. \quad (\text{A.5})$$

In Pauli-Villars regularization, which appears more physical in the present context, one has

$$I_n = \frac{1}{(4\pi)^2} \sum_{k=1}^3 c_k M_k^n \ln\left(\frac{M_k^2}{M^2}\right), \quad n = 0, 2, 4. \quad (\text{A.6})$$

Three regulator masses M_k , of order of the cutoff Λ , are required here with coefficients c_k satisfying $\sum_{k=1}^3 c_k M_k^n + M^n = 0$, for $n = 0, 2, 4$.

(A.4) is now seen to be the flat space expansion to 2nd order of the gravitational action

$$\mathcal{L} = \sqrt{-g} N \left[I_4 + \frac{1}{6} I_2 R - \frac{1}{20} I_0 (R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2) \right] \quad (\text{A.7})$$

with the metric expressed in terms of the vierbein⁴

$$g^{\mu\nu} = e_a^\mu e^{a\nu}, \quad e_a^\mu = \delta_a^\mu + h_a^\mu, \quad (\text{A.8})$$

and

$$h^{\mu\nu} = \delta_a^\mu h^{a\nu}. \quad (\text{A.9})$$

Taking $h^{\mu\nu}$ to be symmetric, as done in the above calculation, amounts to the (standard) local Lorentz gauge fixing to a symmetric vierbein. (It is known that the antisymmetric part in fact decouples in (A.7).) Thus our effective gravitational action (3.1) is reproduced to this order.

To see how this comes about, note that the result (A.4) is precisely what one obtains after integrating out the fermion fields in the Lagrangian for N fermions in curved space:

$$\mathcal{L} = e^{(1-2w)} \left[e_a^\mu \bar{\psi}_i \frac{i}{2} \left(\gamma^a \vec{\nabla}_\mu - \overleftarrow{\nabla}_\mu \gamma^a \right) \psi_i - M \bar{\psi}_i \psi_i \right], \quad (\text{A.10})$$

⁴ Curved and flat indices are denoted by Greek and Latin letters, respectively.

to 2nd order in the expansion about flat space (A.8). In (A.10), $e = \det e_{a\mu}$, and

$$\overleftrightarrow{\nabla}_\mu \equiv \overleftrightarrow{\partial}_\mu \mp \frac{i}{2} \omega_{\mu ab} S^{ab} \quad (\text{A.11})$$

with Lorentz generators $S^{ab} = \frac{i}{4}[\gamma^a, \gamma^b]$. The spinor ψ may, in general, be taken to transform under general coordinate transformations as a density of weight w . The result of the explicit computation is in fact found to be independent of w (see, for example, [13]). Indeed, note that, in terms of the functional integration, the factor $e^{(1-2w)}$ can be absorbed by the change of variables: $\psi \rightarrow e^{(1-2w)/2} \psi'$, $\bar{\psi} \rightarrow e^{(1-2w)/2} \bar{\psi}'$. The spin connection $\omega_{\mu ab}$ is given in terms of the vierbein by

$$\begin{aligned} \omega_{\mu ab} &= \frac{1}{2} e_\mu^a (T_{amn} - T_{mna} - T_{nam}) , \\ T_{mn}^a &= (e_m^\rho e_n^\sigma - e_n^\rho e_m^\sigma) \partial_\sigma e_\rho^a . \end{aligned} \quad (\text{A.12})$$

(A.12) implies that the connection terms in (A.10), in the expansion (A.8), do not give a $h\bar{\psi}\psi$ vertex, but only ‘seagull’ $h^2\bar{\psi}\psi$ and higher h powers vertices. This is most easily seen by rewriting the connection terms in (A.10), after a little rearrangement, in the form

$$e_m^\mu \bar{\psi} \frac{1}{4} \omega_{\mu ab} \epsilon^{mabc} \gamma_c \gamma^5 \psi .$$

Furthermore, it follows from this form that all fermion 1-loop diagrams with two external h -legs do not receive any contribution from the spin connection interaction. The result to $\mathcal{O}(h^2)$ thus agrees with that obtained from (A.3).

The vertices from the spin connection terms will, however, contribute to the diagrams with three or more external legs, reproducing (A.7) to all orders, as dictated by the general coordinate invariance of (A.10). It may therefore appear that, in addition to (A.2), one would need an infinite set of different operators⁵ from (A.1), with precisely specified coefficients, to be included in (A.3) in order to generate (A.7). This, however, is not the case. In the context of (A.1), the connection arises naturally when (A.3) is extended to include the set of powers of the operator

$$\mathcal{O}_\mu^{\kappa\lambda} = \frac{i}{4N} \bar{\psi}_i \{ \gamma_\mu, [\gamma^\kappa, \gamma^\lambda] \} \psi_i , \quad (\text{A.13})$$

⁵ Note that (A.12) contains both $e^{a\mu}$ and its inverse.

in addition to those of (A.2). Introducing the corresponding independent auxiliary field $\omega_{\kappa\lambda}^\mu$, (A.3) is extended to

$$\begin{aligned} \mathcal{L}_{\psi,h} = & (\eta^{\mu\nu} + h^{\mu\nu}) \bar{\psi}_i \frac{i}{2} \left(\gamma_\mu \overrightarrow{\partial}_\nu - \gamma_\mu \overleftarrow{\partial}_\nu + \frac{1}{8} \omega_{\nu\kappa\lambda} \{ \gamma_\mu, [\gamma^\kappa, \gamma^\lambda] \} \right) \psi_i \\ & - M \bar{\psi}_i \psi_i - N \frac{\Lambda^4}{4\pi^2} V(h, \omega) . \end{aligned} \quad (\text{A.14})$$

The fermionic part of (A.14) is equivalent to (A.10) in the first order (Palatini) formulation.

Now, in the first order formalism, the nonpropagating connection in (A.10) serves as a constraint field enforcing vanishing of torsion $e_m^\mu \mathcal{O}^{mab}$ generated by the fermions.

(A.14) differs from (A.10) by the presence of a potential in ω . Hence, variation of the connection will not imply vanishing torsion, and ω will not be expressible entirely in terms of the vierbein as in (A.12).⁶ For illustration purposes, we may adopt a model where the quadratic terms in the potential for ω are suppressed. Then, upon integrating out the fermions, (A.7) and the effective gravitational action (3.1) are reproduced to within small deviations. This is easily seen to be stable under radiative corrections from graviton loops.

It is perhaps worth pointing out again that, as we saw, just the fermionic self-interactions of products of (A.2) in flat space-time already suffice to fully reproduce to second order the Einstein-Hilbert (plus R^2 terms) parts in our effective gravitational action (3.1), i.e reproduce the full content of linearized GR.

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⁶ This is analogous to considering (A.10) in the first order formalism not just by itself, but with the addition of the Einstein-Hilbert action. The latter provides a potential for the connection ω , thus leading to the usual result of torsion generated by fermions. This is of course the usual situation. In the above we were lead to consider (A.10) by itself in the context of generating the Einstein-Hilbert action from (A.1).

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